A Model for the Susceptibility Assessment of Glacial Lake Outburst Floods Based on Physical Processes and the Analytic Hierarchy Process Based on Physical Processes

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Abstract

Objective: The study aims to develop an objective and developable Glacial Lake Outburst Floods (GLOFs) susceptibility assessment model based on physical processes and the Analytic Hierarchy Process (AHP), to quantitatively and objectively evaluate the threats of GLOFs, transforming the errors caused by expert experience in traditional models into errors caused by the quality of objective input data.

Methods: The research introduces the concept of "instantaneous triggering body mechanical energy transfer efficiency" from the perspective of mechanical energy transfer and transformation. Impact indicators are categorized into three major groups, and mechanical energy is used to decouple and assign values to each indicator. An E/F calculation model is constructed, which aligns with the safety factor concept in reliability theory, and the model is validated as a susceptibility assessment tool using the AHP method. The study also summarizes the Analytic Hierarchy Process Based on Physical Processes from the model construction and applies it to build a landslide susceptibility model, thereby verifying its generalization capability.

Results: The study finds that the new model can effectively assess the susceptibility of Glacial Lake Outburst Floods and minimize the influence of subjective experience, offering a novel perspective to understand the dynamic changes of such events. Key indicators include the mechanical energy of instantaneous triggering bodies, the mechanical energy of lake water, and the critical failure Newton force of the dam body. Analysis indicates that the model is highly universal in assessing susceptibility and provides a more comprehensive and systematic framework compared to traditional methods. The Analytic Hierarchy Process Based on Physical Processes demonstrates a certain level of generalization ability.

Limitations: Potential limitations of the study include the difficulty in accurately obtaining some parameters and the need for further validation of the generalization ability of the Analytic Hierarchy Process Based on Physical Processes. Additionally, the accuracy of the model may be constrained by data quality and monitoring technology.

Conclusions: The model proposed in this study has good applicability and universal significance, capable of providing a scientific basis for disaster prevention and mitigation. Compared to existing research, the uniqueness of this work lies in its combination of quantitative methods based on physical processes with AHP, offering a new tool and analytical method for more accurately identifying and assessing disaster risks. The Analytic Hierarchy Process Based on Physical Processes allows the traditional AHP to break free from the limitations of subjective experience.

Keywords: Glacial Lake； Outburst；Analytic Hierarchy Proces； Susceptibility； Quantitative

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1. Introduction

As a sensitive indicator of global change, the cryosphere is not only changing rapidly and significantly but also responding directly and sensitively to the climate system (Qin et al., 2018). Glaciers are an important part of the cryosphere, and under the background of global warming, their melting speed has significantly accelerated (Marta, S. et al., 2021; Lee, E. et al., 2021; Carrivick, J. L. et al., 2023). This phenomenon leads to a large amount of glacial meltwater converging, thereby promoting the further expansion of the existing glacial lake areas. Suitable topographic conditions also contribute to the formation of new unstable glacial lakes, and this trend has continued in recent years (Nie et al., 2017; Harrison et al., 2018; Shugar, D. H et al., 2020). The latest research data shows that there are currently more than 110,000 glacial lakes worldwide, covering an area of about 15,000 square kilometers. From 1990 to 2020, the area of these lakes increased by about 22% (Zhang et al., 2024). In addition, global warming also promotes the frequent occurrence of extreme weather events, which may further intensify the occurrence of ice/snow avalanches (Richardson et al., 2000; Nie et al., 2018), debris flows (Richardson et al., 2000; Haeberli et al., 2017), and floods (Emmer, 2013; Emmer, 2017) that trigger Glacial Lake Outburst Floods. Glacial Lake Outburst Floods disasters have become a major obstacle to the economic and social development of high-altitude and high-cold areas, historically causing tens of thousands of deaths and a large amount of infrastructure damage (Carey, 2005; Liu, J et al. 2014; Allen et al., 2016; Carrivick and Tweed, 2016; Nie et al., 2018). The high-altitude location of glacial lakes means that the floods released during a breach carry huge potential energy and can carry a large amount of material along the way, forming destructive high-sediment floods or debris flows (Cui et al., 2003; Worni et al., 2014; McKillop and Clague, 2007). At present, more than 10 million people worldwide live under the potential threat of Glacial Lake Outburst Floods, which poses a severe challenge to the infrastructure construction and planning in the high mountain Asia region, restricting the sustainable economic development of the area (Xu, 1988; Clague et al., 2000; Sattar et al., 2022; Nie et al., 2023). For countries where the hydropower industry accounts for a large proportion of national economic income, the impact of Glacial Lake Outburst Floods disasters is particularly severe (Carrivick and Tweed, 2016). With the expansion of glacial lake areas and the increase in their numbers, and the increasing frequency of human activities in the areas affected by glacial lakes, the impact of Glacial Lake Outburst Floods disasters is showing an increasing trend, making the analysis of Glacial Lake Outburst Floods susceptibility particularly critical (Gardelle, J et al., 2011; Carrivick, J. L et al., 2013; Nie et al., 2023). This analysis helps us to better grasp the dynamic changes of glacial lake threats (Emmer et al. 2018) and provides a scientific basis and guidance for disaster prevention and mitigation.

2. Literature review

The prediction of Glacial Lake Outburst Floods is significantly different from traditional flood frequency analysis methods in hydrology, mainly because Glacial Lake Outburst Floods often occur as one-time events (Steijn, H. V., 1996; Hegglin, E., 2008). Predicting this phenomenon is a complex scientific issue involving high nonlinearity and uncertainty, requiring consideration of the complex dynamic relationships between inputs and outputs, thereby increasing the difficulty of predicting Glacial Lake Outburst Floods (Zhou, B et al., 2023). To this end, the academic community has developed various models in the assessment of Glacial Lake Outburst Floods susceptibility. Based on the composition of the assessment model, the selection of assessment indicators, and the degree of subjectivity in the calibration of the importance of assessment indicators, these models can be roughly divided into qualitative, semi-quantitative, and quantitative categories (Clague, J.J et al., 2000; Wang, X et al., 2007), where qualitative models are the most subjective, semi-quantitative models are less subjective, and quantitative models are the least subjective. Qualitative models rely heavily on the evaluator's experience and knowledge, generalizing the assessment indicators based on subjective experience, and the threshold settings

of the assessment indicators are also entirely based on subjective experience, resulting in vague and subjective susceptibility results (Carey, M., 2005; Costa, J.E et al., 1988; Huggel, C et al., 2004). Compared to qualitative models, semi-quantitative models have quantified the assessment indicators based on subjective experience, and the use of certain calculation methods has reduced the dependence on the evaluator's experience to a certain extent (Reynolds, J. M., 2003; Bolch, T et al., 2011). Although quantitative models seem more objective in terms of indicator weight allocation and calculation process with the help of certain tools, they are still influenced by subjective experience when selecting assessment indicators and calculation tools (Mckillop R J et al., 2007; Wang, W et al., 2011; Mergili, M et al., 2011). In addition, existing models have not fully considered the coupling effects between different indicators, which may have an adverse impact on the accuracy of the prediction results (Zhou, B et al., 2023). The results of these models are often limited to regional areas, lacking universality, resulting in a situation where a model only works in specific areas (Jiajia Gao et al., 2023), and different evaluators get different grading results for the same area, causing trouble for government decision-making. Models based on the Analytic Hierarchy Process (Nitesh Khadka et al., 2021; Zhang, D et al., 2023), fuzzy comprehensive evaluation method (Wang, W et al., 2011), logistic regression method (Mckillop R J et al., 2007), and fuzzy matter-element extension method (Liu, J.F et al., 2012) play a key role in specific situations, but they are detached from the physical process of Glacial Lake Outburst Floods and cannot reflect the changes in the Glacial Lake Outburst Floods susceptibility index in a refined and dynamic manner. The Glacial Lake Outburst Floods susceptibility index should change with environmental changes, but the indices calculated by existing models do not match their physical characteristics.

3. Theoretical Foundation

3.1 Glacial Lake Outburst Floods Process

A deep understanding of the Glacial Lake Outburst Floods process is the first step in constructing this model. The outburst of a glacial lake is the result of the long-term interaction between the lake water and the ice-dammed barrier, catalyzed by continuous climatic factors such as temperature and precipitation (IPCC, 2013; Harrison et al. 2018). It is triggered by factors such as earthquakes, ice/snow avalanches, landslides, upstream floods/debris flows, and the melting of ice within the dam (Richardson and Reynolds, 2000; Liu Jing-Jing et al. 2014; Worni et al., 2014), leading to partial spontaneous/non-spontaneous destruction of the dam that allows continuous water flow, resulting in erosion and the release of physical energy by the lake water (Liu, J et al. 2013; Westoby et al. 2014; Robin Neupane et al. 2019). Generally, a Glacial Lake Outburst Floods is a dynamic cascading process involving the conversion of potential and kinetic energy (Somos-Valenzuela et al., 2016; Cui, P et al., 2019). It is important to note that the dam's destruction under the action of a surge wave is not necessarily the weakest part of the dam. Due to the varying energy of the surge wave and the differing resistance of various parts of the dam, the point of dam failure is where the energy of the surge wave (lake water) just causes the dam to fail. To achieve a breach, this point must also have the potential to cause continuous overflow of the dam in a relatively short time, leading to the phenomenon of breaching. In fact, what plays a role in the Glacial Lake Outburst Floods is the mechanical energy acting precisely at the point of dam failure and the resistance of the dam at this point. Therefore, identifying the possible points of dam failure is crucial for improving the accuracy of the assessment model. Although we currently cannot precisely predict the exact point of failure, through empirical analysis, we can estimate the possible areas of failure, thereby enhancing the accuracy of the Glacial Lake Outburst Floods susceptibility index solution.

3.2 Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP), proposed by American operations researcher Thomas L. Saaty in the early 1970s, is a method specifically designed to address complex decision-making

problems involving multiple objectives, criteria, factors, and levels (Saaty, T. L., 1980). The general process of this method begins by breaking down the decision-making problem into the objective level, criterion level, and alternative level, forming a hierarchical structure model. Immediately following, at the criterion and alternative levels, each element is compared pairwise, and scores are assigned based on relative importance. Then, the pairwise comparison matrix is subjected to a consistency check to ensure the rationality of the evaluation. Next, the weight vector is calculated through the pairwise comparison matrix, which usually involves the computation of eigenvalues and eigenvectors. Finally, the weights of the criterion level are combined with the weights of the alternative level to calculate the comprehensive score of the alternatives and rank them (Saaty, T. L., 1990; Saaty, T. L., 1994; Saaty, T. L., 2008; Saaty, T. L. and Vargas, L. G., 2001; Saaty, T. L., 2005).

The core of the AHP lies in its hierarchical decision-making structure, the assessment and quantification of importance, and the calculation and synthesis of weights. Even if there are changes in computational details or technical means, as long as these core steps are retained, the method still conforms to the spirit of the AHP. For example, mathematical tools such as fuzzy logic (Zadeh, L. A., 1965; Dubois, D. & Prade, H., 1980) and the ideal point method (Tzeng, G. H. & Huang, J. J., 2011) can be combined to assist in determining weights or consistency checks.

The significance of hierarchization is that it allows complex decision-making problems to be divided into different levels according to logic or criteria, making it easier to understand and handle problems more systematically. Weight synthesis allows decision-makers to consider the impact of multiple criteria on alternatives comprehensively and to determine the relative importance of each criterion. Through synthesis, a comprehensive evaluation result can be obtained. This method not only provides a way to quantify comparisons and select the best alternative but also helps decision-makers make more rational and scientific choices when facing complex decision-making problems.

Due to its ability to unify qualitative and quantitative factors in decision-making, the AHP has been widely applied in various fields. In the assessment of Glacial Lake Outburst Floods susceptibility, the AHP has also been utilized. Wang et al. (2011) developed a first-order method based on the AHP to identify potentially dangerous glacial lakes in the southeastern Tibetan Plateau. Five variables were selected: the area of the parent glacier, the distance between the lake and the glacier terminus, the slope between the lake and the glacier, the average slope of the moraine dam, and the steepness of the glacier terminus. Weights for these variables were assigned using the Fuzzy Consistent Matrix (FCM) method. Then, characteristic statistical values were used as thresholds for classifying each variable, successfully identifying 8 glacial lakes with extremely high risk of outburst from 78 moraine-dammed lakes. In addition, the method was validated by using 6 historical Glacial Lake Outburst Floods. Nitesh Khadka et al. (2021) applied the AHP to assess the susceptibility of Glacial Lake Outburst Floods in the Mahalangur Himalaya, determining the weights of six key factors such as lake area, expansion rate, distance from the ice lake, dam front slope, ice/snow avalanche potential, and upstream GLOF potential through expert pairwise comparison, and then calculating the susceptibility index to classify the ice lakes into different levels of susceptibility from very low to very high. Zhang, D et al. (2023) combined the AHP with digital elevation models (DEM), glacier data, remote sensing images, and field surveys to establish an assessment method for quantifying the susceptibility of Glacial Lake Outburst Floods floods (GLOF) in the Nam Co Basin of the northern Tibetan Plateau, successfully identifying and verifying high-risk glacial lakes.

3.3 Safety Factor

In the construction of the model for analyzing and calculating the susceptibility index of individual Glacial Lake Outburst Floods, the concept coincides with the relevant ideas of the safety factor in reliability theory. The application of reliability theory in the safety assessment of existing

structures has been widely recognized (Diamantidis, D et al., 2024). When we discuss the safety of existing structures, reliability theory provides a powerful analytical framework, especially by introducing the core concept of the safety factor. The definition of the structural safety factor (Safety Factor, SF) may vary slightly across different engineering fields and standards, but the core concept is similar. The safety factor is usually defined as the ratio of the load-bearing capacity considered in the design of a structure or component to its actual load-bearing demand under the most unfavorable load combination (ISO 13822:2017; IBC 2018). This factor is used to ensure that the structure can safely withstand various anticipated and unexpected loads during its designed service life. In traditional engineering practice, the safety factor is usually defined as the ratio of the existing strength parameters to the strength parameters required to maintain stability (Song, E.X et al., 2016). Glacial lakes, as products of nature, consist of components such as instantaneous triggering bodies, the glacial lake itself, and the dam. In the process of Glacial Lake Outburst Floods, these components work together to form a kind of natural existing structural system. The concept of the safety factor should also apply to this analysis.

4. Model Construction

4.1 Analysis and Construction of the Index System

From a practical standpoint, achieving absolute accuracy in obtaining the susceptibility index of Glacial Lake Outburst Floods is unrealistic. Our goal is to get as close as possible to the susceptibility index of Glacial Lake Outburst Floods. To achieve this, an accurate understanding of random factors and other influencing factors is required. However, due to limitations in technical means, access to information resources, levels of knowledge and theory, as well as natural environmental conditions, human understanding of all factors affecting Glacial Lake Outburst Floods is limited. Especially for complex influencing factors, the limitations in technical understanding and the uncertainties in subjective understanding are particularly evident. This paper only seeks to improve the understanding of the susceptibility index of Glacial Lake Outburst Floods under existing conditions.

Therefore, starting from the perspective of mechanical energy transfer and transformation, we have identified the three main participants directly involved in the process: instantaneous triggering bodies, the glacial lake itself, and the dam body. The instantaneous triggering bodies include ice/snow avalanches, landslides, debris flows, and floods, etc., and the glacial lake itself includes two parts: the lake water and the lake basin. All indicators affecting the Glacial Lake Outburst Floods directly or indirectly act on these three main bodies, thereby participating in the Glacial Lake Outburst Floods and thus affecting the susceptibility index of the Glacial Lake Outburst Floods.

For the instantaneous triggering bodies, the indicators related to their impact on the susceptibility index of Glacial Lake Outburst Floods can be divided into two parts: one is the indicators affecting their occurrence probability, and the other is the indicators affecting the calculation of their mechanical energy entering the lake. The indicators affecting the occurrence probability include air temperature, precipitation, earthquakes, and their own innate endowments, etc. The indicators affecting the calculation of mechanical energy entering the lake include the mass, shape, density, volume, distance from the lake, height from the lake, and friction coefficient of the path into the lake of the instantaneous triggering bodies, etc. For the glacial lake itself, the indicators related to the susceptibility index of Glacial Lake Outburst Floods are mainly divided into two parts: indicators related to the lake basin and indicators related to the lake water. The indicators related to the lake basin include the shape, slope, and volume of the lake basin; the indicators related to the lake water include the volume, density, and composition distribution of the lake water, etc. The resistance of the dam body is also crucial, and the indicators affecting the susceptibility index of Glacial Lake Outburst Floods are also divided into two parts: indicators related to the shape of the dam body and indicators related to the internal composition of the dam body. The indicators

related to the shape of the dam body include the height, width, and upstream slope of the dam body; the indicators related to the internal composition of the dam body include the particle size distribution, mineral composition, structural distribution of the dam soil, and the content of ice in the dam body, etc. The indicators related to the instantaneous triggering bodies and the glacial lake itself are mainly used for energy and probability calculations, while those related to the dam body are used for resistance calculations.

In previous assessments of the susceptibility of Glacial Lake Outburst Floods, the above indicators were often not fully considered as assessment indicators. However, as the physical properties of the main bodies involved in the Glacial Lake Outburst Floods, these indicators cannot be ignored in any assessment model. For example, the area of the lake water, the rate of change of the lake water area, and the ratio of the depth of the lake water to the height of the dam, which are commonly used for the assessment of the susceptibility of Glacial Lake Outburst Floods, can all be regarded as representations of the lake water, and ultimately reflect the state of the mechanical energy of the lake water at the dam break location; similarly, the area, thickness, distance from the lake, and development of cracks of the rear edge glacier, which are commonly used indicators, can all be regarded as representations of the instantaneous triggering body, and ultimately reflect the calculation of the mechanical energy entering the lake and the probability of occurrence of the instantaneous triggering body. Furthermore, from the perspective of the transfer and transformation of mechanical energy, we have identified some new indicators, such as the distance of the instantaneous triggering body relative to the dam break location, the angle of the instantaneous triggering body entering the lake, the wind speed when the instantaneous triggering body moves, and the number of blocks when the instantaneous triggering body disintegrates into the lake, etc., which help to more accurately calculate the mechanical energy when the instantaneous triggering body enters the lake.

4.2 Construction of Energy Transfer Efficiency

Since the instantaneous triggering bodies typically do not make direct contact with the dam's failure location, the mechanical energy of the instantaneous triggering bodies is transmitted to the dam's failure location in the form of a surge wave through the glacial lake water. We consider the energy of the surge wave acting at the dam's failure location to be the energy contributed by the instantaneous triggering bodies to the Glacial Lake Outburst Floods. Therefore, we define the ratio of the mechanical energy of the lake water at the dam's failure location caused by the instantaneous triggering bodies to the mechanical energy of the lake water by the instantaneous triggering bodies as the energy transfer efficiency.

$$\eta = \frac{E_b}{E_h} \tag{1}$$

In the formula, η represents the energy transfer efficiency, $E_b$is the mechanical energy of the lake water caused by the instantaneous triggering body at the dam's failure location, and E_h is the mechanical energy of the lake water by the instantaneous triggering body.

4.3 Construction of Dam Body Resistance

Glacial lakes are categorized based on their dam types into ice dams, moraine dams, and rock dams (Otto, J.C, 2019). During the process of Glacial Lake Outburst Floods, the dam body may be subjected to various forces, including tensile stress that leads to rupture failure, shear force that causes shear failure, seepage force that results in seepage damage, and buoyancy generated by surge waves against dams, etc. The modes of failure also exhibit diversity, which may include scour failure, seepage failure, rupture failure, and shear failure, etc.

The complexity of dam failure stems from a variety of factors, including the diversity of failure modes, the diversity of forces experienced, the complexity of the properties of the moraine soil material, the irregularity of the dam body's geometric shape, and the randomness of the interaction

between the surge wave and the dam, etc. These factors collectively determine that the critical Newton force required for the failure of the dam cannot be accurately calculated, meaning that the resistance of the dam is unsolvable. However, despite the inability to obtain an exact value, resistance still has expressiveness.

There are many factors affecting the unsolvable nature of the dam body's resistance, which can mainly be summarized as the performance of the dam body's constituent materials, the geometric parameters of the dam body, and the calculation model. Material properties involve physical characteristics such as its strength, elastic modulus, and Poisson's ratio. Due to differences in the composition and structure of the dam body materials, as well as the influence of environmental conditions, the performance of the dam body material may vary. The geometric parameters of the dam body, such as height, width, length, and slope, may also change under the influence of the environment. These changes may have a minor impact on the resistance of the moraine dam in the short term and are usually treated as constant values in calculations. The calculation of the dam body's resistance is usually based on some basic assumptions, which may not fully conform to the actual situation, or the calculation formula itself has approximations, thus introducing variability. Despite this, what we can determine is that there is a specific critical Newton force when the dam body fails. This force can be represented as the product of stress and area, where stress represents the material properties, and area represents the geometric parameters of the dam body. Through the critical Newton force, we can characterize the overall resistance of the dam body at the failure location.

$$F = \delta cs \tag{2}$$

In the formula, δ represents the ratio of the actual dam resistance to the calculated resistance; c indicates the shear strength of the dam soil, which is used to characterize the performance of the dam material; s denotes the shear area of the dam, which is used to represent the geometric parameters of the dam; F signifies the critical failure Newton force of the dam, which is used to represent the overall resistance of the dam body.

4.4 Selection, Decoupling, and Valuation of Indicators

The paper has analyzed and constructed important parameters of the index system involved in Glacial Lake Outburst Floods, which can be summarized into three major indicators: indicators affecting the mechanical energy contributed by the instantaneous triggering body to the dam's failure location, indicators affecting the mechanical energy contributed by the lake water to the dam's failure location, and indicators affecting the resistance at the dam's failure location. As also mentioned in the research status, existing models do not fully decouple the interactions between indicators, and the selection and weighting of assessment indicators are still influenced by subjective experience. This section addresses these issues.

Since the Glacial Lake Outburst Floods is a dynamic cascading process involving the transfer and transformation of mechanical energy, which runs through the entire process of the outburst, we have taken mechanical energy into account when building the model. Mechanical energy serves two purposes here: one is to value the indicators, and the other is to decouple the interactions between various indicators. When calculating the mechanical energy contributed by the instantaneous triggering body to the dam's failure location and the mechanical energy contributed by the lake water to the dam's failure location, we found that they are independently calculated and do not interfere with each other. The result reflected in the model is that the weight of different indicators is the amount of mechanical energy they contribute—the larger the value, the greater the weight. This naturally values the two major indicators of mechanical energy contributed by the instantaneous triggering body to the dam's failure location and the lake water to the dam's failure location, and solves the coupling problem and valuation problem between indicators. This paper constructs the model from the perspective of mechanical energy transfer and transformation, and the division and selection of indicators based on objective physical processes include all

influencing factors without any subjective experience. The resistance at the dam's failure location does not belong to the link of mechanical energy transfer and transformation, so it is not of the same category as the other two types of indicators and naturally does not couple with other indicators. However, the resistance at the dam's failure location is also an important indicator affecting the Glacial Lake Outburst Floods, and its value also affects the solution and analysis of the Glacial Lake Outburst Floods susceptibility index. Valuing it is also necessary, and here we value the resistance at the dam's failure location with the critical failure Newton force.

4.5 Model Proposal

After an in-depth analysis of the physical processes and theoretical foundations of Glacial Lake Outburst Floods, we have divided the three main participants in the Glacial Lake Outburst Floods system into two main categories: the attacking party and the defending party. The attacking party includes the instantaneous triggering bodies and the glacial lake itself, which contribute mechanical energy to the dam's failure location through the instantaneous triggering bodies and the lake water contributes mechanical energy to the dam's failure location; the defending party refers to the dam body itself, whose defensive capability is manifested in the form of the critical failure Newton force at the dam's failure location. Thereupon, we have constructed a model for the analysis and solution of the Glacial Lake Outburst Floods susceptibility index. This model aims to consider various factors in the Glacial Lake Outburst Floods system comprehensively and to analyze and solve the Glacial Lake Outburst Floods susceptibility index in a quantitative manner.

$$GLOFSI=\frac{\sum_{i=j=k=1}^{n}\psi_i\eta_j E_{hk}+G_b}{F*1m}=\frac{\sum_{i=j=k=1}^{n}\psi_i\eta_j E_{hk}}{F*1m}+\frac{G_b}{F*1m} \tag{3}$$

$$E_h=E_{all}-E_s \tag{4}$$

In the formula, the *GLOFSI* (Glacial Lake Outburst Floods Flood Susceptibility Index) represents the susceptibility index of Glacial Lake Outburst Floods floods. We are only concerned with the specific numerical value of this ratio, so it is divided by 1 meter to eliminate the units, resulting in a dimensionless number; ψ represents the probability of occurrence of the instantaneous triggering body, with a maximum value of 1 and a minimum value of 0; η represents the energy transfer efficiency, with a maximum value of 1 and a minimum value of 0; E_h represents the mechanical energy of the lake water by the instantaneous triggering body; G_b represents the potential energy of the lake water at the dam's failure location; E_{all} represents the initial potential energy of the instantaneous triggering body; E_s represents the mechanical energy lost by the instantaneous triggering body before entering the lake; F represents the critical failure Newton force at the dam's failure location, which is used to represent the overall resistance of the dam body; here, the unit of energy is joules, and the unit of force is newtons。

If the denominator directly used the critical failure energy at the selected location, a dimensionless number could be directly obtained, but we have chosen the critical failure Newton force to represent the dam's resistance. This decision is based on two main reasons:

First, compared to the critical failure energy, the critical failure Newton force is more easily obtained through technical means; both c and s can be measured, with only δ being an empirical coefficient. However, estimating the critical failure energy involves too many uncertain parameters, leading to significant estimation errors. Second, just like the critical failure energy, the critical failure Newton force can strictly represent the dam's resistance, and it exists under all circumstances. For the above considerations, this paper has selected the critical failure Newton force to characterize the dam's resistance. However, if other forms (including the critical failure

energy) are used to represent the calculation model of the dam's resistance, similar to the E/F calculation model constructed here, it remains a valid calculation model.

4.6 Model Analysis

When quantitatively analyzing the susceptibility index of individual Glacial Lake Outburst Floods, we find that all influencing factors can be directly or indirectly reflected in the corresponding susceptibility expressions of the participating entities, and they collectively determine the changes in the Glacial Lake Outburst Floods susceptibility index. Importantly, the changes in the Glacial Lake Outburst Floods susceptibility index do not always correspond to dramatic changes in individual parameters; instead, a comprehensive assessment from a systemic perspective is required. For instance, if the cracks in the dangerous ice avalanche body at the rear edge of a glacier have increased, enhancing the probability of its occurrence, but at the same time, the volume of the glacial lake water has significantly decreased due to evaporation and infiltration, we cannot solely attribute the increased probability of ice avalanches to a rise in the Glacial Lake Outburst Floods susceptibility index, nor can we attribute the decrease in lake water to a reduction in the susceptibility index. It is necessary to consider the changes in the remaining parameters of the expression. Moreover, changes in factors that we subjectively believe to influence may not necessarily alter the Glacial Lake Outburst Floods susceptibility index, as these changes must result in alterations of the parameters within the expression to affect the susceptibility index. This systemic perspective provides a more comprehensive understanding of the changes in the susceptibility index of individual Glacial Lake Outburst Floods.

Each glacial lake has its unique natural characteristics and conditions, and therefore, the susceptibility index expression for each Glacial Lake Outburst Floods is established based on its specific situation. When analyzing the susceptibility index of an individual Glacial Lake Outburst Floods, the first step is to identify and understand the natural endowment of the glacial lake, including its geographical location, morphology, hydrological conditions, etc., and to determine the relevant parameters and their values as accurately as possible. Based on these parameters, we can construct and analyze a susceptibility index expression specific to that Glacial Lake Outburst Floods. By analyzing the expression of the Glacial Lake Outburst Floods susceptibility index in the form of a ratio, we can observe that within a certain period, if the numerator (representing the energy of the attacking party) decreases while the denominator (representing the resistance of the defending party) increases, the Glacial Lake Outburst Floods susceptibility index will decrease; conversely, if the numerator increases while the denominator decreases, the Glacial Lake Outburst Floods susceptibility index will rise. When both the numerator and the denominator change simultaneously, the trend of change in the Glacial Lake Outburst Floods susceptibility index may not be so apparent. In such cases, if we can obtain the specific extent of the changes in the numerator and the denominator through monitoring or other means, we can more accurately determine the changes in the Glacial Lake Outburst Floods susceptibility index. This is derived from the analysis of the concept of ratio.

The expression for the Glacial Lake Outburst Floods susceptibility index is also composed of addition, where the numerator is split into two parts, each divided by the denominator to yield a sum of two parts, each representing a different concept. The first part represents the Glacial Lake Outburst Floods susceptibility index contributed by the instantaneous triggering body, and the second part represents the Glacial Lake Outburst Floods susceptibility index contributed by the potential energy of the lake water at the dam's failure location. In other words, the Glacial Lake Outburst Floods susceptibility index consists of two parts: one contributed by the instantaneous triggering body and the other by the lake water. When analyzing the susceptibility index of an individual Glacial Lake Outburst Floods, if both parts of the susceptibility index increase, the

overall susceptibility index also increases; if both parts decrease, the overall susceptibility index decreases as well. When one part of the susceptibility index increases and the other decreases, it is then necessary to resolve the susceptibility index for the corresponding part, but current technical means cannot achieve precise valuation. At this point, we can use quantitative analysis to determine which part plays a dominant role in the susceptibility index. Often, the changes in the part that plays a dominant role determine the changes in the Glacial Lake Outburst Floods susceptibility index, especially when the change in the dominant part's susceptibility index is greater than that of the non-dominant part. In some cases, although there may be errors in judging the changes in the Glacial Lake Outburst Floods susceptibility index, when the changes in the dominant and non-dominant parts are not easily distinguishable, that is, when the change in the dominant part's susceptibility index is small and the change in the non-dominant part's susceptibility index is large, the overall change in the Glacial Lake Outburst Floods susceptibility index caused by the combined changes will not be significant.

Specifically, for moraine-dammed lakes without instantaneous triggering bodies, we can construct their Glacial Lake Outburst Floods susceptibility index expressions and change discriminants to analyze the changes in the susceptibility index under specific environmental changes.

$$GLOFSI=\frac{G_b}{F*Im} \quad (5)$$

When $\frac{\Delta G_b * F}{\Delta F * G_b}=1$, The susceptibility index of Glacial Lake Outburst Floods remains unchanged; (6)

When $\frac{\Delta G_b * F}{\Delta F * G_b}>1$, the susceptibility index of Glacial Lake Outburst Floods increases; (7)

When $\frac{\Delta G_b * F}{\Delta F * G_b}<1$, the susceptibility index of Glacial Lake Outburst Floods decreases. (8)

For instance, under conditions of rising temperatures, the meltwater from glaciers increases, but changes in precipitation and evaporation infiltration may be uncertain. Concurrently, the critical failure Newton force at the dam's failure location may decrease due to the melting of buried ice and frozen debris. Under such circumstances, we generally anticipate an increase in the susceptibility index of Glacial Lake Outburst Floods. However, according to our model of the Glacial Lake Outburst Floods susceptibility index, a more precise assessment of changes in the volume of glacial lake water is required, which can be achieved by monitoring the water level corresponding to the assumed dam breach range. It is also necessary to evaluate whether the resistance at the dam's failure location has decreased or remained almost unchanged. If the water level at the dam's failure location has risen, we can essentially determine that the susceptibility index of the Glacial Lake Outburst Floods has increased. If evaporation and infiltration are too strong, causing the water level at the dam's failure location to drop, then the specific changes in the glacial lake susceptibility index require further analysis. But it can be preliminarily judged that, in this case, the increase in the Glacial Lake Outburst Floods susceptibility index may not be significant, and it may even decrease.

For glacial lakes with only a single hazardous ice avalanche body, establishing an exclusive expression for the Glacial Lake Outburst Floods susceptibility index is crucial. With the

continuous impact of global climate warming, the probability of instantaneous triggering bodies occurring may increase. However, as the ice body continues to melt, its volume and potential energy decrease continuously, with high potential energy gradually transforming into the low potential energy of the lake water. During this mechanical energy transformation process, the parameters involved in the calculation of the Glacial Lake Outburst Floods susceptibility index are also constantly adjusting and changing. It is worth noting that in this process, a phenomenon where the Glacial Lake Outburst Floods susceptibility index may actually decrease can occur. For example, in the past, ice avalanche bodies entering the lake could always cause a Glacial Lake Outburst Floods, but when the hazardous ice body melts to the point where the surge waves caused by its fall into the lake are not sufficient to trigger an outburst, this phenomenon is proven. This is contrary to intuitive expectations. At the same time, as glaciers gradually melt, the distance between the rear edge of the glacier and the glacial lake becomes increasingly farther, leading to an increase in the mechanical energy consumption of the hazardous ice avalanche body entering the lake and a decrease in the mass of the hazardous ice avalanche body. If the glacier meltwater does not increase the volume of the glacial lake water due to infiltration and evaporation, the decrease in the numerator is greater than the decrease in the denominator, which is also evidence that the susceptibility index of some glacial lakes decreases under global warming.

Previous studies have also considered the issue of the minimum glacial lake area threshold (Nie et al., 2018; Veh et al., 2019). Glacial lakes below a certain area threshold are inevitably not considered by researchers and are usually ignored when interpreting glacial lakes through remote sensing images. However, small-area glacial lakes can also cause "small breach, great disaster" phenomena under the right conditions (Liu, M et al., 2020; Zhang, T et al., 2022). By establishing an expression for the Glacial Lake Outburst Floods susceptibility index, it is not difficult to find that the susceptibility index of the neglected small-area glacial lakes may also be higher than that of large-area glacial lakes, which cannot be captured in some assessment models that overemphasize area and scale as indicators. This provides a clear signal to Glacial Lake Outburst Floods disaster prevention personnel that although the damage caused by small-area Glacial Lake Outburst Floods is not as severe as that caused by large-area Glacial Lake Outburst Floods, when the susceptibility index of small-area glacial lakes is higher than that of large-area glacial lakes, small-area glacial lakes should also be given equal attention. At the same time, we can also find that no matter how small the area of the glacial lake, it has its own susceptibility index expression. That is, the regional Glacial Lake Outburst Floods susceptibility assessment based on this model does not need to consider the issue of the minimum glacial lake area threshold value, indicating that the model's universality has been further strengthened.

Glacial lakes at a lower water level usually have a lower susceptibility index compared to those at a higher water level, which is in line with our subjective understanding and is also reflected in the Glacial Lake Outburst Floods susceptibility index model we have constructed. When at a lower water level, without considering the changes in instantaneous triggering bodies, the mechanical energy contributed by the lake water is relatively small, and the breach location of the dam often moves correspondingly downward. Generally, the lower the dam body, the thicker it is, and the greater the Newton force required for destruction. According to the analysis of the established Glacial Lake Outburst Floods susceptibility index expression, if the numerator decreases and the denominator increases, the result decreases, that is, the Glacial Lake Outburst Floods susceptibility index becomes smaller. At the same time, the possibility of a surge wave breach is smaller because the surge wave needs to break through a thicker area to produce continuous overflow. Therefore, in this case, the main mode of Glacial Lake Outburst Floods destruction is dominated by seepage/pipe flow. Similarly, without considering the changes in instantaneous triggering bodies, as the water level rises, the Glacial Lake Outburst Floods susceptibility index tends to rise accordingly. When the energy of the surge wave can easily break the dam, the mode of Glacial Lake Outburst Floods destruction at this time should be dominated by surge wave breach and seepage/pipe flow. As the water level continues to rise, the Glacial Lake Outburst Floods susceptibility index continues to increase, and overflow scouring also becomes a new dominant

mode of destruction, and the mode of Glacial Lake Outburst Floods destruction at this time should be dominated by overflow scouring, surge wave breach, and seepage/pipe flow.

Through the previous analysis, we can also understand that changes in water level can cause changes in the Glacial Lake Outburst Floods susceptibility index and even determine the mode of dam breach, indicating that the water level plays a controlling role in Glacial Lake Outburst Floods. When the water level is close to zero, the instantaneous triggering body (except for floods/debris flows) entering the lake will not cause a Glacial Lake Outburst Floods; when the water level reaches its maximum value, the glacial lake may easily burst. Between these two extreme water levels, there must be a critical water level at which the instantaneous triggering body falls into the lake without causing a breach, and this water level is referred to as the glacial lake safety water level. Similarly, when the water level of the glacial lake is stable in a certain range for many years, there is also a suitable instantaneous triggering body that does not cause a breach when it falls into the lake. The instantaneous triggering body in this situation is called the safe instantaneous triggering body. These analyses provide new insights for engineering management: in addition to improving the resistance of the dam body, the susceptibility index of individual glacial lakes can also be effectively regulated by lowering the water level to the safety water level, reinforcing the instantaneous triggering body, reducing the mass of the instantaneous triggering body, or controlling the water level, instantaneous triggering body, and dam body simultaneously. This kind of control is essentially adjusting the parameter values of the influencing factors to reduce the Glacial Lake Outburst Floods susceptibility index and achieve the ideal engineering management effect of the glacial lake, which is in line with our subjective understanding.

This model is entirely based on objective physical processes. Although some parameters may be difficult to obtain accurately due to objective reasons, the model can accurately determine the trend of changes in the susceptibility index of individual Glacial Lake Outburst Floods in some cases. If this model is applied to multiple glacial lakes in a region and the results obtained are ranked, it is possible to assess the regional susceptibility of Glacial Lake Outburst Floods, that is, a model for assessing the susceptibility of Glacial Lake Outburst Floods has been constructed. For conclusions drawn using other evaluation methods, this model can serve as a verification tool under certain conditions.

4.7 Model Explanation

Since this model is constructed based on physical processes, its structural form coincides with that of the safety factor, which can lead to confusion with stability analysis. Therefore, it is necessary to clarify the differences between this model and stability analysis models, as well as the similarities between this model and the Analytic Hierarchy Process (AHP) models, thereby indicating that this model is a susceptibility analysis model rather than a stability analysis model.

In this paper, the three main entities are divided into the attacking and defending parties. The attacking and defending parties are in the same Glacial Lake Outburst Floods system, so they have equal importance but cancel each other out. In the basic arithmetic operations, it is obviously not appropriate to add or multiply the representative quantities of the two parties. This leaves only the subtraction and division of the two parties' representative quantities. When the attacking party's representative quantity is subtracted from the defending party's representative quantity, we find that the outburst susceptibility index of large-scale glacial lakes is generally higher than that of small-scale glacial lakes, which is obviously not in line with common sense. This leaves only the division of the two, so we use the division of the two parties' representative quantities to construct the model and find that it meets the initial expectation. In this way, the structural form of the division of the two parties' representative quantities is similar to that of the safety factor, but this structural form is different from that of the safety factor. The safety factor is usually a dimensionless quantity, while the E/F calculation mode adopted by this structure obtains a quantity with units. Since only the numerical value is of interest, it is divided by 1 meter to eliminate the

units. If this structure adopts the E/E calculation mode, a dimensionless number can also be directly obtained. In summary, this model does not need to maintain the same dimensions for the numerator and the denominator, which is essentially different from the safety factor. Moreover, the numerical value sought by this model is much larger than the conventional value of the safety factor. This value often cannot explain the problem by itself and only makes sense when compared with the calculated values of other glacial lakes (including those that have already burst) or with its own previous values. It can also be said that the inverse calculation mode of the safety factor is a special case of the susceptibility index calculation mode, which requires the numerator and the denominator of the susceptibility index calculation mode to have the same dimensions (E/E calculation mode, F/F calculation mode) and does not include probability items (such as the probability of occurrence of instantaneous triggering bodies). At the same time, the safety factor is usually defined as the ratio of the load-bearing capacity considered in the design of a structure or component to its actual load-bearing demand under the most unfavorable load combination. The most obvious difference from the Glacial Lake Outburst Floods susceptibility index expression is that it does not include probability items (such as the probability of occurrence of instantaneous triggering bodies) and the Glacial Lake Outburst Floods susceptibility index expression is the ratio of the load side to the resistance side. The above is sufficient to illustrate that this model is not conducting a safety factor analysis (stability analysis).

It was also mentioned earlier that the core of the AHP lies in its hierarchical decision-making structure, the assessment and quantification of importance, and the calculation and synthesis of weights. Even if there are changes in computational details or technical means, as long as these core steps are retained, the method still conforms to the spirit of the AHP. The entire Glacial Lake Outburst Floods susceptibility index solution analysis model is also very similar to the composition of the AHP. When applying the AHP to this model, the target layer is the susceptibility of Glacial Lake Outburst Floods, the criterion layer is the value of the three major indicators: the value of the mechanical energy contributed by the instantaneous triggering body to the dam's failure location, the value of the mechanical energy contributed by the lake water to the dam's failure location, and the value affecting the resistance at the dam's failure location. The scheme layer is the susceptibility index of each glacial lake to be evaluated. It shows that the hierarchical decision-making structure is met. We find that the value of the mechanical energy contributed by the instantaneous triggering body to the dam's failure location and the value of the mechanical energy contributed by the lake water to the dam's failure location belong to the same category. They are both representative quantities of the attacking party, and their units are consistent, so they can be directly added and summed after being calculated separately. In this way, there are only two types of indicators, one is the representative quantity of the attacking party - potential energy, and the other is the representative quantity of the defending party - the critical failure Newton force. After classifying the various influencing factors of the Glacial Lake Outburst Floods susceptibility index based on the physical process and quantifying and solving the three major indicators, the assignment of each indicator has been completed, which also means that the model does not need to perform a consistency check. When constructing the comparison matrix for the criterion layer and the scheme layer respectively, especially since the two types of indicators belong to different categories, when constructing the matrix in the criterion layer, the representative quantities of the attacking and defending parties should be separated and their judgment matrices should be constructed separately. What's more special is that after the value of each glacial lake indicator to be evaluated is solved, it objectively exists and is independent of each other, without any subjective judgment. Therefore, each glacial lake to be evaluated can have its own judgment matrix (the purpose of the judgment matrix in the criterion layer is to measure the relative importance of each indicator and assign corresponding weights), and there is no need to construct a judgment matrix that all Glacial Lake Outburst Floods susceptibility index calculations follow through subjective experience. In the end, the pairwise comparison matrix of the criterion layer is two 1×1 matrices. The construction of the scheme layer's judgment matrix is similar to the ordinary AHP. In this way, the weights of the criterion layer and the scheme layer

can be determined. The two types of indicators are classified based on the physical process and follow the physical relationship (E/F calculation mode), which just serves the problem of the target layer, so this calculation mode needs to be followed when calculating and synthesizing the weights. At the same time, the impact of each indicator on the Glacial Lake Outburst Floods susceptibility index can be comprehensively reflected in the calculation expression. It is surprising to find that the calculation results obtained by directly bringing the values of each indicator into the calculation mode and the results obtained by the AHP mode are still in a multiple relationship, which is realized by assuming data entry.
Hypothetical Data:
Glacial Lake 1 has a mechanical energy of 8 joules (J) and a resistance of 2 newtons (N); Glacial Lake 2 has a mechanical energy of 12 joules (J) and a resistance of 6 newtons (N); Glacial Lake 3 has a mechanical energy of 15 joules (J) and a resistance of 3 newtons (N).

1、Directly solve by substituting into the calculation model：

Glacial Lake 1： $GLOFSI = \frac{8J}{2N \times 1m} = 4$；

Glacial Lake 2； $GLOFSI = \frac{12J}{6N \times 1m} = 2$；

Glacial Lake 3： $GLOFSI = \frac{15J}{3N \times 1m} = 5$；

2、The following is the process of solving by the Analytic Hierarchy Process (AHP):

(1) Construct the pairwise comparison matrix for mechanical energy at the criterion level.

Glacial Lake 1	Mechanical Energy
Mechanical Energy	1
Glacial Lake 2	Mechanical Energy
Mechanical Energy	1
Glacial Lake 3	Mechanical Energy
Mechanical Energy	1

（2）Construct the pairwise comparison matrix for the Resistance Force at the criterion level.

Glacial Lake 1	Resistance Force
Resistance Force	1
Glacial Lake 2	Resistance Force
Resistance Force	1

Glacial Lake 3	Resistance Force
Resistance Force	1

(3) Construct the pairwise comparison matrix for mechanical energy at the alternative level.

Mechanical Energy	Glacial Lake 1	Glacial Lake 2	Glacial Lake 3
Glacial Lake 1	1	2/3	8/15
Glacial Lake 2	3/2	1	4/5
Glacial Lake 3	15/8	5/4	1

(4) Construct the pairwise comparison matrix for the Resistance Force at the alternative level.

Resistance Force	Glacial Lake 1	Glacial Lake 2	Glacial Lake 3
Glacial Lake 1	1	1/3	2/3
Glacial Lake 2	3	1	2
Glacial Lake 3	3/2	1/2	1

(5) Calculation and Synthesis of Weights

Glacial Lake 1 Mechanical Energy weight:

$$\frac{1+2/3+8/15}{(1+2/3+8/15)+(3/2+1+4/5)+(15/8+5/4+1)}=8/45$$

Glacial Lake 1 Resistance Force weight:

$$\frac{1+1/3+2/3}{(1+1/3+2/3)+(3+1+2)+(3/2+1/2+1)}=2/11$$

Integrated calculation of weights from the criterion layer and the alternative layer:

$$GLOFSI = \frac{8/45\times 1}{2/11\times 1} = 44/45$$

Similarly, the following can be obtained:

Glacial Lake 2： $GLOFSI = \dfrac{12/45\times 1}{6/11\times 1} = 22/45$

Glacial Lake 3； $GLOFSI = \dfrac{15/45\times 1}{3/11\times 1} = 55/45$

By directly substituting the indicator values into the calculation model and comparing the results with those obtained after constructing the pairwise comparison matrix through the normal Analytic Hierarchy Process (AHP), it is found that they are in a multiple relationship. The value of this multiple is always equal to the sum of the mechanical energy of each glacial lake divided by the sum of the resistance forces of the dams. At the same time, for the sake of simplicity in calculation, when calculating the pairwise comparison matrix at the criterion layer, it is usually only necessary to construct a pairwise comparison matrix for a single glacial lake.

From this, we can conclude that the model can directly omit the step of constructing the comparison matrix at the criterion and scheme layers, and there is no need to consider consistency checks. Finally, the calculation results of the glacial lakes that have already burst and those to be evaluated can be compared and sorted, and can also be compared with their own previous glacial lake susceptibility indices. After comparison with other glacial lakes to be evaluated at the same time, intervals can be divided for susceptibility assessment, and the change in the Glacial Lake Outburst Floods susceptibility index can be analyzed by comparing it with its own previous Glacial Lake Outburst Floods susceptibility index. In theory, all samples of glacial lake bursts can serve as a historical database, and the Glacial Lake Outburst Floods susceptibility assessment model will no longer be restricted by regions, further strengthening the model's universality. The above is sufficient to illustrate that it is a model that naturally embeds physical processes into the Analytic Hierarchy Process without considering subjective experience, and it is also a susceptibility assessment model.

5. Construction Method of the Analytic Hierarchy Process Based on Physical Processes

The construction and result derivation of the Glacial Lake Outburst Floods susceptibility assessment model appear very natural, with no involvement of subjective experience. For now, let's call the method of constructing the Glacial Lake Outburst Floods susceptibility assessment model based on physical processes the Analytic Hierarchy Process Based on Physical Processes. The Glacial Lake Outburst Floods susceptibility assessment model based on physical processes can be said to be a typical application of the Analytic Hierarchy Process Based on Physical Processes. Therefore, it is necessary to summarize the commonality of the Analytic Hierarchy Process Based on Physical Processes from the applied model. Thus, I will summarize and refine the process of constructing the Glacial Lake Outburst Floods susceptibility assessment model to further introduce the Analytic Hierarchy Process Based on Physical Processes.

The first step in constructing the Analytic Hierarchy Process Based on Physical Processes is to clarify our target layer, and then to deeply understand the physical processes (Glacial Lake Outburst Floods) involved in the problem of the target layer (susceptibility of Glacial Lake Outburst Floods). Next, classify the factors affecting the problem of the target layer to establish the criterion layer (the value of the mechanical energy contributed by the instantaneous triggering body to the dam's failure location, the value of the mechanical energy contributed by the lake water to the dam's failure location, and the value affecting the resistance at the dam's failure location), and clarify the physical relationships between the classified categories in service of the target layer problem (attacker and defender), and establish the physical relationship (E/F calculation model in the Glacial Lake Outburst Floods susceptibility model). It is essential to find the flow quantity (quantify each indicator with flow quantity, with energy as the flow quantity in the Glacial Lake Outburst Floods susceptibility model). Then establish the scheme layer (susceptibility indices of each glacial lake).

Next, construct the pairwise comparison matrices for the criterion layer and the scheme layer separately, noting that the pairwise comparison matrices for the numerator and denominator of the criterion layer need to be constructed separately. Each individual to be evaluated has its own pairwise comparison matrix. Since as a fraction, the units of each term in the numerator must be

consistent and can generally be merged into one item, the same applies to the denominator, so the pairwise comparison matrix of the criterion layer is two 1×1 matrices. This 1×1 matrix is also a comparison with oneself. The method of constructing the pairwise comparison matrix for the scheme layer is consistent with the method of constructing the pairwise comparison matrix in the ordinary hierarchy method.

Finally, the weights are integrated and calculated based on the physical relationships. After meeting the requirements of the target layer problem, these results are ranked, divided, or compared with the previous susceptibility indices of the individuals to be evaluated. It is found that when the values assigned to each indicator are directly substituted into the calculation model, and the results are in a multiple relationship with those obtained by normally constructing the pairwise comparison matrix (this step can be achieved by assuming multiple sets of data), the model is essentially complete. Thus, the Analytic Hierarchy Process based on physical processes is constructed.

5.1 Validation Method of the Analytic Hierarchy Process Based on Physical Processes: Landslide Susceptibility Analysis Model Based on Physical Processes

The Glacial Lake Outburst Floods susceptibility model based on physical processes is a typical application of the Analytic Hierarchy Process (AHP) based on physical processes. Through the construction, analysis, and refinement of the previous model, it is not difficult to find that the AHP based on physical processes, as a method, should have a certain generalization ability. Therefore, this paper finally attempts to establish a landslide susceptibility assessment model based on physical processes.

At this time, the target layer is the susceptibility of landslides; the criterion layer is the potential energy of the slope body at the foot of the slope, the additional instantaneous mechanical energy, and the anti-sliding force of the slope; the scheme layer is the susceptibility index of each slope body. By clarifying the physical relationships of various indicators at the criterion layer, it is found that they still follow the E/F calculation model, as shown in Equation (9). Then, construct the pairwise comparison matrix for the criterion layer and the scheme layer. After the two parts of the numerator are merged, the pairwise comparison matrix constructed by the numerator and the denominator for the criterion layer is still a 1×1 matrix. The pairwise comparison matrix for the scheme layer is constructed according to the normal AHP. The results obtained by directly substituting the values of the indicators into the calculation model and those obtained through normal hierarchical analysis are still in a multiple relationship, which is achieved by assuming multiple sets of data. Obtain the susceptibility indices of various landslides (including the susceptibility indices of landslides that have already occurred), and then compare these results with each other to rank and divide the high and low susceptibility intervals.Or these results can be compared and analyzed with their previous indices.

$$LSI = \frac{G + \sum_{i=j=1}^{n} \psi_i E_{fj}}{F * 1m} \tag{9}$$

In the formula, *LSI* refers to the landslide susceptibility index. We are only concerned with the specific numerical value of this ratio, so it is divided by 1 meter to eliminate the units and obtain a dimensionless number; *G* represents the potential energy of the slope body at the foot of the slope, Ψ is the probability coefficient of the instantaneous additional mechanical energy taking effect, with a maximum value of 1 and a minimum value of 0; E_f is the instantaneous additional mechanical energy; *F* is the anti-sliding force of the slope body; the unit of energy is joules, and the unit of force is newtons.

By constructing the landslide susceptibility assessment model, it has been proven that the Analytic Hierarchy Process based on physical processes has a certain generalization capability. It also builds a bridge between stability analysis and susceptibility analysis. We often believe that the more unstable landslides there are in a region, the higher the landslide susceptibility in that area tends to be. This issue is explained to some extent here. When the probability item coefficient Ψ is determined to be 0 or 1 and adopts the *E/E* calculation mode, *F/F* calculation mode, the more unstable the landslide, the greater the landslide susceptibility index obtained from the calculation formula. If there are more unstable landslides in the region, then there are more landslides with an increased landslide susceptibility index, and at this time, the regional landslide susceptibility is higher.

6. Conclusion

This paper introduces an innovative model for assessing the susceptibility of Glacial Lake Outburst Floods, which relies minimally on subjective experience and is deeply rooted in physical processes, solvable and analyzable by the Analytic Hierarchy Process (AHP). Although newly proposed, the model is inherently complex, especially in terms of accurately obtaining its many parameters. Pursuing precise parameter estimation is not only a scientific pursuit but also a key step in enhancing the model's predictive capabilities and practical application value, marking it as an important direction for the future development of this model.

The model's construction is structurally similar to the assessment of safety factors, aiming to reflect the comparison between offensive forces (such as the mechanical energy of triggering events and lake water) and defensive forces (represented by the dam's resistance). This comparison is not just a theoretical construct but is key to a comprehensive and systematic assessment of the susceptibility index.

An important direction of this research is the application of AHP to physical processes, which has been proven by the successful application of the physical process-based AHP to models assessing both Glacial Lake Outburst Floods and landslide susceptibilities. This physical process-based hierarchical analysis method allows the traditional AHP to break free from the limitations of subjective experience. This versatility indicates that the physical process-based AHP has the potential to extend beyond the current domain, demonstrating its applicability in a broader range of areas.

However, the development of the model is not without obstacles. The challenge of accurately determining certain parameters, such as the exact location of dam failures, underscores the need for ongoing refinement. Nonetheless, the model's application is unrestricted by regional limitations and is not confined by minimum glacial lake area thresholds, with no subjective experience involved from beginning to end.

In summary, the model proposed in this paper represents a significant advancement in enhancing our ability to assess the susceptibility of Glacial Lake Outburst Floods. By integrating physical processes with a hierarchical decision-making framework, it not only lays a solid foundation for subsequent research but also provides practical applications for disaster prevention and mitigation efforts in mountainous regions.

Note: The model is in its initial stage and is ongoing work that requires rigorous validation and iterative enhancement. We extend an open invitation to scholars and practitioners to contribute their insights and suggestions, with the shared goal of improving the model's accuracy and broadening its practical relevance.

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